

Assumptions: Random Sample

$$np_0, nq_0 \geq 5$$

Section 8.3

Name _____

SHORT ANSWER. Write your answer in the space provided or on a separate sheet of paper.

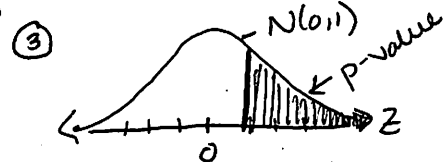
Identify the null hypothesis, alternative hypothesis, test statistic, P-value, conclusion about the null hypothesis, and final conclusion that addresses the original claim.

- 1) In a clinical study of an allergy drug, 108 of the 202 subjects reported experiencing significant relief from their symptoms. At the 0.01 significance level, test the claim that more than half of all those using the drug experience relief.

Given: $x=108$, $n=202$, $\hat{p} = x/n = 108/202 = 0.5347$

$\alpha = 0.01$, $p_0 = 0.50$

① $H_0: p = 0.50$ *
 $H_A: p > 0.50$ (claim)



$$\begin{aligned} P(Z > 0.99) &= 1 - P(Z < 0.99) \\ &= 1 - 0.8389 \\ &= \boxed{0.1611} \end{aligned}$$

② $Z_s = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.5347 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{202}}} = \boxed{0.99}$

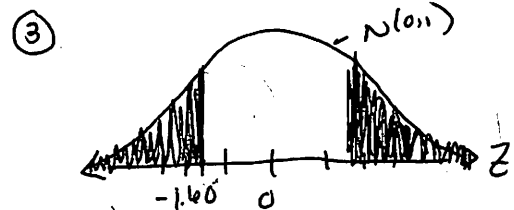
Our sample value ($\hat{p} = 0.5347$) is 0.99 st. deviations above our hypothesized value of 0.50.

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- 2) According to a recent poll 53% of Americans would vote for the incumbent president. If a random sample of 100 people results in 45% who would vote for the incumbent, test the claim that the actual percentage is 53%. Use a 0.10 significance level.

Given: $n=100$, $\hat{p}=0.45$, $p_0=0.53$, $\alpha=0.10$

① $H_0: p = 0.53$ (claim) *
 $H_A: p \neq 0.53$
 (< or >)



② $Z_s = \frac{0.45 - 0.53}{\sqrt{\frac{0.53 \times 1 - 0.53}{100}}} = -1.60$

$$\begin{aligned} 2 \cdot P(Z < -1.60) &= 2 \cdot (0.0548) \\ &= \boxed{0.1096} \end{aligned}$$

Our sample value (0.45) is 1.60 standard deviations below the hypothesized value of 0.53.

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Problem 1

Interpret : If H_0 were true, the true proportion of subjects experiencing relief is 0.50, then we would see our data 16.11% of the time.

④ a) Since $p\text{-value} = 0.1611 > \alpha = 0.01 \rightarrow \text{FTR } H_0$.

b) At 1% sig. level, there is not sufficient evidence to support the claim that more than half of subjects experience relief.

Problem 2

Interpret : If H_0 were true, the true proportion of Americans who would vote for the incumbent president is 53%, then would observe our data 10.96% of the time.

④ a) Since $p\text{-value} = 0.1096 > \alpha = 0.10 \rightarrow \text{FTR } H_0$.

b) (The claim is in H_0 - the book uses "warrant the rejection of" in the conclusion)

At 10% sig level, there is not sufficient evidence to warrant the rejection of the claim that the true proportion of Americans who would vote for the incumbent president is 0.53.

Testing a Claim About a Mean of Two INDEPENDENT Populations:
 σ_1 and σ_2 are NOT Known (Section 9.3)

Notation:

- μ_1 = population mean of the first sample
- σ_1 = population standard deviation of sample 1
- $\bar{x}_1$ = mean of sample 1
- s_1 = standard deviation of sample 1
- n_1 = Size of the first sample

- μ_2 = population mean of the second sample
- σ_2 = population standard deviation of sample 2
- $\bar{x}_2$ = mean of sample 2
- s_2 = standard deviation of sample 2
- n_2 = Size of sample 2

When we do not know the population standard deviation σ , we must use the Student t distribution instead of the Normal Distribution.

The Student t distribution is displayed in Table A-3 of your textbook.

Requirements: (Assumptions)

- The two samples are INDEPENDENT.
- Both samples are simple random samples.
- The values of either population standard deviations are NOT known. (s_1, s_2)
- Both populations are normally distributed or both have size greater than 30. (CLT)

Degrees of freedom for inferences about 2 means: Independent Samples

- Our textbook uses:

Degrees of freedom = smaller of $n_1 - 1$ and $n_2 - 1$

- Your TI 83/84 calculator (most accurate estimate) uses:

$$\text{Degrees of freedom} = \frac{(A + B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}}$$

$$\text{where } A = \frac{s_1^2}{n_1} \text{ and } B = \frac{s_2^2}{n_2}$$

- Other textbooks/references may use other formulas such as

$$\text{Degrees of freedom} = n_1 + n_2 - 2$$

Hypotheses

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

$$H_0: \mu_1 = \mu_2$$

$$H_A: \mu_1 \neq \mu_2$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 > 0$$

$$H_0: \mu_1 = \mu_2$$

$$H_A: \mu_1 > \mu_2$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 < 0$$

$$H_0: \mu_1 = \mu_2$$

$$H_A: \mu_1 < \mu_2$$

Test Statistic

$$t_s = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

df = smaller of $n_1 - 1$
 $n_2 - 1$

Example:

Many studies have been conducted to test the effects of marijuana use on mental abilities. In one such study, groups of light and heavy users of marijuana in college were tested for memory recall, with the results given below. Use a 0.01 significance level to test the claim that the population of heavy marijuana users has a lower mean than the light users. Should marijuana use be of concern to college students?

$$\alpha = 0.01 \quad df = 64 - 1 = 63$$

- (1) Items sorted correctly by light marijuana users: $n = 64, \bar{x} = 53.3, s = 3.6$
(2) Items sorted correctly by heavy marijuana users: $n = 65, \bar{x} = 51.3, s = 4.5$ $\downarrow$ - table!

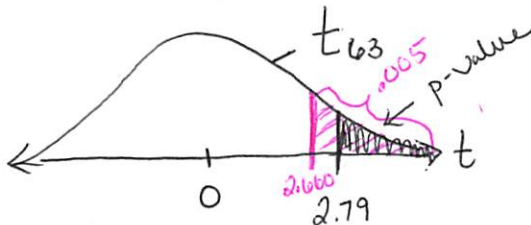
$$\begin{aligned} \textcircled{1} \quad H_0: \mu_1 &= \mu_2 & H_0: \mu_1 - \mu_2 &= 0 \\ H_A: \mu_1 &> \mu_2 \text{ (claim)} * & H_A: \mu_1 - \mu_2 &> 0 \end{aligned}$$

$$\textcircled{2} \quad t_s = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(53.3 - 51.3) - 0}{\sqrt{\frac{3.6^2}{64} + \frac{4.5^2}{65}}} = 2.79$$

The sample value $(\bar{x}_1 - \bar{x}_2)$ is 2.79 st. deviations above the hypothesized difference of 0.

- ③ Estimate the p-value.

Table: Look at $df = 63$, use $df = 60$.



$$P(t > 2.660) = 0.005$$

$$p\text{-value} < 0.005$$

If H_0 were true, there is no difference in memory recall for heavy vs. light users, I would observe my data less than 0.5% of the time.

④ a) Since p-value $< .005$ $< \alpha = 0.01 \rightarrow$ Reject H_0 .

b) At the 1% sig level, there is sufficient evidence to support the claim that heavy users have a lower mean recall than light users.

Confidence Interval : We can construct CI's to make a decision regarding the claim for two-sided tests, since CI's are naturally two-sided, ^($H_a: \neq$)

Interpretation : $LB < \mu_1 - \mu_2 < UB$

1) If the bounds for the differences are positive, then $\mu_1 - \mu_2 > 0 \rightarrow \mu_1 > \mu_2$

2) If the bounds are negative, then $\mu_1 - \mu_2 < 0 \rightarrow \mu_1 < \mu_2$

3) If the bounds cover 0, then $\mu_1 - \mu_2 = 0 \rightarrow \mu_1 = \mu_2$

Example:

Listed below are the numbers of years that popes and British monarchs (since 1690) lived after their election or coronation. Treat the values as simple random samples from a larger population. Use a 0.01 significance level to test the claim that the mean longevity for popes is less than the mean for British monarchs after coronation.

$$\alpha = 0.01$$

(1) Popes:

2 9 21 3 6 10 18 11 6 25 23 6
2 15 32 25 11 8 17 19 5 15 0 26

$$n_1 = 24 \quad s_1 = 8.96$$

$$\bar{x}_1 = 13.13$$

(2) Kings and Queens:

17 6 13 12 13 33 59 10 7 63 9 25 36 15

$$n_2 = 14 \quad s_2 = 18.60$$

$$\bar{x}_2 = 22.71$$

$$df = 14 - 1 = 13$$

$$\textcircled{1} H_0: \mu_1 = \mu_2$$

$$H_a: \mu_1 < \mu_2 \text{ (claim)}$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 - \mu_2 < 0 \text{ (claim)}$$

$$\textcircled{2} t_s = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \underline{-1.81}$$

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Popes Example

Assumptions

Independent Samples,

Random Samples,

σ_1 and σ_2 unknown

where $\sigma_1 \neq \sigma_2$

degrees of freedom

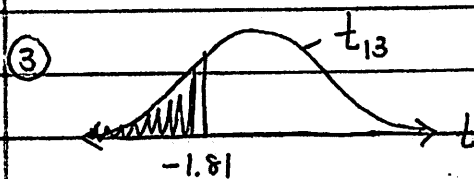
$$df = n_1 - 1 = 24 - 1 = 23$$

$$n_2 - 1 = 14 - 1 = 13$$

① $H_0: \mu_1 = \mu_2$ *

$H_A: \mu_1 < \mu_2$ (claim)

② $t_s = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(13.13 - 22.71) - 0}{\sqrt{\frac{8.96^2}{24} + \frac{18.60^2}{14}}} = -1.81$



Find: $P(t < -1.81) = P(t > 1.81)$

From Table: @ $df = 13$

$P(t > 1.771) = 0.05$, $P(t > 2.160) = 0.025$

Since $1.771 < 1.81 < 2.160$, we

can say $0.025 < p\text{-value} < 0.05$

Interpretation of t_s and p-value:

t_s : The sample value $(\bar{x}_1 - \bar{x}_2)$ is an estimated
1.81 standard deviations away from the true hypothesized
difference of 0.

p-value: If H_0 were true, the mean longevity of Popes is the same as Monarchs, then we would observe our data between 2.5% and 5% of the time.

④ Since p-value $> \alpha = 0.01 \rightarrow$ FTR H_0

⑤ At the 1% significance level, there is not sufficient evidence to support the claim that the mean longevity for Popes is less than the British Monarchs.

Inferences from DEPENDENT Samples (Section 9.4)

Notation:

- d = difference between a matched pair
- μ_d = mean value of the differences of the population
- $\bar{d}$ = mean value of the differences for the paired sample data (sample difference)
- s_d = standard deviation of the differences for the paired sample data
- n = number of pairs of data

Paired Data : Before/After , Husband/Wife , Mother/Daughter

Use the t -distribution, since we do not know σ .

The test statistic is:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

→ claimed value of differences
($\mu_d = 0$ for our class)

$$H_0: \mu_d = 0$$

$$H_A: \mu_d \neq 0$$

$$H_0: \mu_d = 0$$

$$H_A: \mu_d > 0$$

$$H_0: \mu_d = 0$$

$$H_A: \mu_d < 0$$

$$(\mu_d = \mu_1 - \mu_2)$$

$\left\{ \begin{array}{l} \mu_d \text{ is the notation used for the} \\ \text{dependent case.} \end{array} \right\}$

$$df = \# \text{ pairs} - 1$$

Example: Listed below are measured fuel consumption amounts (in miles per gallon) from a sample of cars. Use a 0.05 significance level to test the claim that there is no difference in fuel consumption between city and highway driving.

$$\alpha = 0.05, df = 4 - 1 = 3$$

Fuel Consumption	
Car	City/Highway
Acura RL	18/26
Acura TSX	22/31
Audi A6	21/29
BMW 525i	21/29

d (city-highway)

-8
-9
-8
-8

$$\bar{d} = \frac{1}{4} [-8 + -9 + -8 + -8] = -8.25$$

$$s_d = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = 0.5$$

Dependent Data

① $H_0: \mu_d = 0$ (claim)

$H_a: \mu_d \neq 0$ *
(two-tailed)

② $t_s = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{-8.25 - 0}{0.5 / \sqrt{4}} = -33$

The sample value ($\bar{d}$) is an estimated 33 st. deviations below the hypothesized difference of 0.

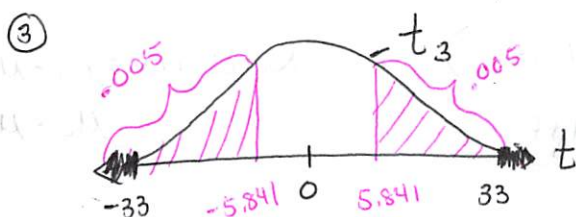


Table: $df = 3, 2 \cdot P(t > 33) = 0.01$

$p\text{-value} < 0.01$

If H_0 is true, there were no difference in fuel consumption between city/highway, we would observe our data less than 1% of the time.

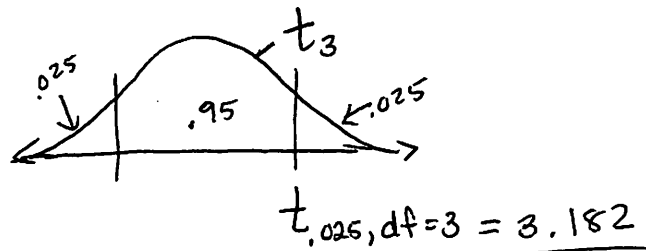
④ a) Since $p\text{-value} < 0.01 < \alpha = 0.05 \rightarrow \text{Reject } H_0$.

b) At 5% sig level, there is sufficient to warrant the rejection of the claim that there is no difference in city/highway mpg.

Note: This was a two-sided test. We can construct a CI.

$$\bar{d} \pm t_{\alpha/2} (s_d / \sqrt{n}) \quad df = n - 1 = 4 - 1 = 3$$

$$\begin{aligned} 1 - \alpha &= 0.95 \\ \alpha &= 0.05 \\ \alpha/2 &= 0.025 \end{aligned}$$



$$-8.25 \pm 3.182 (0.5 / \sqrt{4})$$

$$(-9.05 < \mu_d < -7.45)$$

diff diff

Note: $\mu_d = \mu_1 - \mu_2$
 $\mu_c - \mu_h$

I am 95% confident the true difference in city vs. highway mpg is between -9.05 and -7.45.

Furthermore, since bounds are negative, $\mu_c - \mu_h < 0$.

$\rightarrow \mu_c < \mu_h$. Further evidence to reject H_0 since

the CI did not cover 0. (Note: $\mu_d < 0$)

Example: Listed below are the ages of actresses and actors at the times they won Oscars. The data are paired according to the years they won. Use a 0.05 significance level to test the common belief that best actresses are younger than best actors. Does the result suggest a problem in our culture?

↳ actresses - actors = negative

(1) Best Actresses: $\begin{matrix} \text{yr} & \text{yr} \\ 28 & 32 & 27 & 27 & 26 & 24 & 25 & 29 & 41 & 40 & 27 & 42 & 33 & 21 & 35 \end{matrix}$

(2) Best Actors: 62 41 52 41 34 40 56 41 39 49 48 56 42 62 29

$\begin{matrix} d \\ -34 & -9 & -25 & -14 & -8 & -16 & -31 & -12 & 2 & -9 & -21 & -14 & -9 & -41 & 6 \end{matrix}$
Given: $\bar{d} = -15.667$ $sd = 12.877$

* Actresses - Actors ($\mu_1 - \mu_2$) Note here we say μ_d

① $H_0: \mu_d = 0$
 $H_A: \mu_d < 0$ (claim)

② $t_s = \frac{\bar{d} - \mu_d}{sd/\sqrt{n}} = \frac{-15.667 - 0}{12.877/\sqrt{15}} = -4.71$

The sample difference ($\bar{d} = -15.667$) is an estimated 4.71 st. deviations below the hypothesized difference of 0.

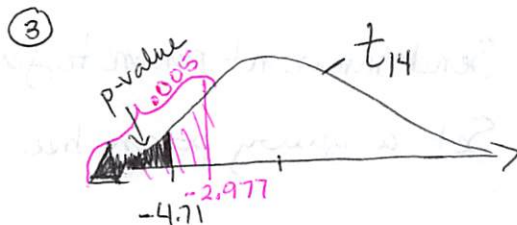


Table: $P(t > 2.977) = 0.005$
 $P(t < -2.977) = 0.005$ (symmetry)

$\boxed{p\text{-value} < .005}$

If H_0 were true, there was no difference in age between actors and actresses at the time they win Oscars, we would observe our data less than 0.5% of the time.

④ a) Since $p\text{-value} < .005 < \alpha = 0.05 \rightarrow \text{Reject } H_0$.

b) At 5% level of significance, there is sufficient evidence to support the claim that actresses are younger than actors at the time they won Oscars.

Type I and Type II Error

Reality	Decision	
	Reject H_0	FTR H_0
H_0 True	Type I Error	Correct
H_0 False	Correct	Type II Error

$$P(\text{Type I Error}) = P(\text{Reject } H_0 \mid H_0 \text{ True}) = \alpha \quad *$$

$$P(\text{Type II Error}) = P(\text{FTR } H_0 \mid H_0 \text{ False}) = \beta$$

As α increases, β decreases. As α decreases, β increases.

Ex: H_0 : Innocent
 H_a : guilty

Type I Error: Send innocent person to jail

Type II Error: Set a guilty person free.

For the actress/actors examples: I could have made Type I Error because I Reject H_0 .

Interpret: I concluded actresses were younger than actors, when in reality, actresses and actors are approximately the same age.

Example: Data below includes measured weights of 5 college students in September and in April of their freshman year. Use a 0.05 level of significance to test the claim that for the mean change in weight for the population of students from September to April is equal to 0 kg.

Student 1: April weight = 66 kg September weight = 67 kg

Student 2: April weight = 52 kg September weight = 53 kg

Student 3: April weight = 68 kg September weight = 64 kg

Student 4: April weight = 69 kg September weight = 71 kg

Student 5: April weight = 71 kg September weight = 70 kg

practice

Answer on
next page →

Example: Data below includes measured weights of 5 college students in September and in April of their freshman year. Use a 0.05 level of significance to test the claim that for the mean change in weight for the population of students from September to April is equal to 0 kg.

Student 1:	April weight = 66 kg	September weight = 67 kg	$\frac{d}{-1}$
Student 2:	April weight = 52 kg	September weight = 53 kg	-1
Student 3:	April weight = 68 kg	September weight = 64 kg	4
Student 4:	April weight = 69 kg	September weight = 71 kg	-2
Student 5:	April weight = 71 kg	September weight = 70 kg	1

$$\bar{d} = \frac{(-1) + (-1) + 4 + (-2) + 1}{5} = 0.2$$

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{4} [(-1-0.2)^2 + (-1-0.2)^2 + (4-0.2)^2 + (-2-0.2)^2 + (1-0.2)^2]} = 2.387$$

① $H_0: \mu_d = 0$ (claim) *
 $H_A: \mu_d \neq 0$

④ Since $p\text{-value} > 0.20 > \alpha = 0.05$
 $\rightarrow$ FTR H_0 .

② $t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{0.20 - 0}{2.387 / \sqrt{5}} = 0.187$

Our sample value, $\bar{d}$, is an estimated 0.187 st. deviations above our hypothesized difference of 0.

⑤ With 95% confidence, there is sufficient evidence to support the claim that the mean change in weight for the students is equal to 0.

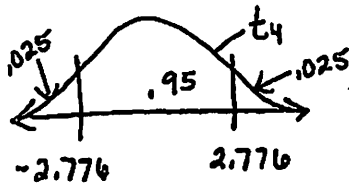


2
 Table: At $df=4$, find value(s) closest to 0.187.
 $P(t > 1.533) = 0.20 \rightarrow \boxed{p\text{-value} > 0.20}$

Example: Data below includes measured weights of 5 college students in September and in April of their freshman year. Use a 0.05 level of significance to test the claim that for the mean change in weight for the population of students from September to April is equal to 0 kg.

Student 1:	April weight = 66 kg	September weight = 67 kg
Student 2:	April weight = 52 kg	September weight = 53 kg
Student 3:	April weight = 68 kg	September weight = 64 kg
Student 4:	April weight = 69 kg	September weight = 71 kg
Student 5:	April weight = 71 kg	September weight = 70 kg

Construct a 95% confidence interval.



$$\bar{d} = 0.2 \quad s_d = 2.387 \quad n = 5$$

Table: $df = 4, \alpha = 0.05$ (two tails)

$$\bar{d} \pm t_{\alpha/2} (s_d / \sqrt{n}) \rightarrow 0.2 \pm 2.776 (2.387 / \sqrt{5}) \rightarrow (-2.76, 3.16)$$

I am 95% confident the true mean difference in weight is between -2.76 and 3.16. Note, the interval contains the value 0, which implies $\mu_d = 0$ ($\mu_1 - \mu_2 = 0$). Thus, more evidence to fail to reject H_0 , the mean change in weight for the students is 0.